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FOCAL POINT COMPUTATION IN A THREE-LAYERED ATMOSPHERE

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ABSTRACT

This report outlines a computer program for calculating, directly from the atmospheric data, the location of ground level acoustic foci resulting from meteorological conditions in a three-layered atmosphere. Also, a procedure is given whereby one obtains the average intensity level in the immediate neighborhood of a focus. Finally, it is shown, by means of examples, that a systematic use of the computer program can lead to a deepened insight into the relationship of the focus location and the atmospheric parameters.

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TECHNICAL AND SCIENTIFIC STAFF
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SUMMARY

This report outlines a computer program for calculating, directly from the atmospheric data, the location of ground level acoustic foci resulting from meteorological conditions in a three-layered atmosphere. Also, a procedure is given whereby one obtains the average intensity level in the immediate neighborhood of a focus. Finally, it is shown, by means of examples, that a systematic use of the computer program can lead to a deepened insight into the relationship of the focus location and the atmospheric parameters.

I. INTRODUCTION

As a result of the procedures used in obtaining atmospheric data, a velocity-of-sound profile is normally represented by a many-sided polygon, where the velocity is customarily plotted as a function of height. This polygon, corresponding to a multi-layered atmosphere, is assumed to be a more or less accurate approximation of the true vertical velocity distribution. From a given profile one obtains velocities and heights which, in turn, are used as input to the computer program written to calculate sound ray patterns and sound ray landing distances. Normally, the only output of concern is the set of landing distances associated with a given velocity profile, since, from this set, a determination may be made as to the existence (or nonexistence) of one or more focal points. This examination procedure is both laborious and time consuming, and sometimes fails even to indicate the presence of a focus.

It would, therefore, be advantageous to have some rapid method not only of calculating focal distances directly from the atmospheric data but also of insuring that all focal points have been obtained. Such a direct method would be very useful in handling several of the problem areas which exist in sound propagation studies, resulting from the vast amount of output generated by present computer programs. For example, consider the problem of determining the effect on focal distances of variations in the atmospheric parameters. It is almost impossible, using present techniques, to make any sort of comprehensive survey of the relationship between the atmospheric parameters and focal distances because of the overwhelming volume of computer output which would have to be analyzed. Let us consider another problem: Assume that there exists a profile which, contrary to observation, indicates no focus formation when analyzed by existing techniques. One might wish to explore the possibilities of varying the input parameters in various ways calculated to result in a profile which does give focus formation. (However, one is entitled to vary the atmospheric parameters only within the accuracy limits of the instruments which make the meteorological measurements.) This problem can be dealt with, in a reasonable length of time, only by an examination of a number of different velocity profiles, each of which is only slightly different from the original. In addition, one could examine the possibility of varying a profile in some manner in order to obtain a focus at a given location.

A direct method of focal point computation would be of significant use in handling these problems. It could also be useful when one is faced with a problem which requires analyzing a large amount of computer output. However, from both a mathematical and a numerical point of view, such a method is nearly impossible to obtain in the case of a general multi-layered atmosphere. This is explained as follows.

The condition for a focal point is that the equation

$$1 + \sum_{i=1}^{k-1} \left[(D_i - D_{i+1}) \frac{1}{\sqrt{1 + \delta_i q}} \right] = 0 \qquad (1)$$

should be satisfied.* (The D's and δ's are functions of the atmospheric parameters, k is the number of layers in the atmosphere, and q is related to the focal ray's angle of departure, θ_o^*, which is the unknown parameter.) If a q can be found which satisfies equation (1), it can then be used to calculate the focal distance, x_F, from

$$x_F = \frac{2y_1}{k_1 - 1} \frac{1}{\sqrt{q}} \left[1 + \sum_{i=1}^{k-1} \left[(D_i - D_{i+1}) \sqrt{1 + \delta_i q} \right] \right]. \qquad (2)$$

* Expression (1) is the same as equation (55), Ref. 1, p. 40, with $k_i = 1 - \delta_i$, $\zeta/(1 - \zeta) = q$.

Therefore, the problem of direct focal point computation hinges on the ability to solve equation (1), for q.

When k = 2, corresponding to a two-layered atmosphere, equation (1) can be solved quite readily (cf. Ref. 1, pp. 30-39).

When k = 3, corresponding to a three-layered atmosphere, equation (1) is not quite so readily solved for q. For a few values of the D's and δ's, analytic solutions exist, but, in general, q must be found by some numerical methods, the best of which were found to be iterative procedures. A complete mathematical treatment of equation (1), for k = 3, may be found in Reference 2.

When k = 4, corresponding to a four-layered atmosphere, equation (1) can be rewritten as a twelfth-degree polynomial in q. This polynomial not only has no analytical solution, but also is not readily amenable to numerical root-finding techniques because the coefficients are ill-conditioned. When $k > 4$, expression (1) becomes even more difficult to handle.

The mathematical difficulties associated with the multi-layered case prohibit the development of a direct method of focal point calculation, except in the two simple cases where k = 2 and k = 3. Fortunately, many multi-layered situations can be approximated by these simple cases when information is sought as to possible focus formation. This is not to say that the simple cases should be substituted for the more complex cases every time, but, in general, it can be said that if the two- or three-layered approximant provides a focus, the multi-layered case will also.*

Therefore, it was decided to take the mathematical techniques presented in Reference 2 and to develop a computer program for direct focal point computation in the three-layered atmosphere. With the aid of such a program, it would be possible to gain a deepened insight into the relationship between atmospheric parameters and focus formation.

This report, then, describes the computer program and how it can be used. Also given are some preliminary results of investigations carried out in some of the areas discussed previously. A copy of the program, written for the CDC 3200 computer, by Mr. J. Hilliard, is included as Appendix I for those who wish to explore its possibilities further.

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* Reference 5 also points this out.

II. COMPUTER PROGRAM

To facilitate discussion of the computer program, a sample page of computer printout (Table 1) is included here. In general, the computer program allows one to input any set of initial velocities and heights (Table 1, Lines 9-10), any set of increments greater than 1 x 10^{-4} to be applied to these initial values (Table 1, Lines 12-13), and any set of integers, from 1 to 999 (Table 1, Line 7), corresponding to the number of times each increment is to be applied. One other quantity, tolerance 1, (Table 1, Line 4), must be specified as a criterion for the convergence of an iteration procedure. The example shows that only the initial velocity, V_1, will be incremented, by an amount 0.0125 a total of 35 times, with a convergence criterion of 1 x 10^{-7}.

Furthermore, the user has control (by means of two sense switches) of what information is printed out: Sense switch 1 gives the option of printing each step of an iteration (cf. Table 1, Lines 35-49); sense switch 2 gives the option of printing the quantities, exemplified by Table 1, Lines 22-25, computed from the velocities and heights. Normally, the sense switches are off since, if one is running a particularly large number of cases, there is no wish to obtain any more printout than necessary. However, either sense switch may be turned on during execution of the program if the user desires to monitor the progress of the program, or if he desires to verify that everything is being computed properly.

Although the computer program is basically designed to determine foci resulting from atmospheric conditions in the third layer, it also will determine whether or not there are foci resulting from atmospheric conditions in the second layer. This section was included not only for completeness but also to take care of the following eventuality. Suppose that in the course of a systematic survey, where several velocities and heights are being varied simultaneously, it happens that a certain combination of the parameters results in two adjacent atmospheric layers having the same gradient.* When this occurs, one no longer has a three-layered atmosphere but has, instead, an atmosphere of only two layers, which must be handled differently.

Therefore, there has been built into the program existence criteria for foci to result from atmospheric conditions in both the two-layered and three-layered atmospheres.

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* The gradient is defined as the rate of change of propagation velocity of sound with respect to height.

TABLE I

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                              THREE—LAYERED ATMOSPHERE ANALYSIS

                                TOLERANCE1=   1.00000000E-07

 CV0=   1      CV1=  35      CV2=   1      CV3=   1      CY0=   1      CY1=   1      CY2=   1      CY3=   1

V0I= 3.400000000E 02        V1I= 3.395000000E 02        V2I= 3.380000000E 02        V3I= 3.430000000E 02
Y0I=                0        Y1I= 1.000000000E 02        Y2I= 2.500000000E 02        Y3I= 5.000000000E 02

DV0=                0        DV1= 1.250000000E-02        DV2=                0        DV3=                0
DY0=                0        DY1=                0        DY2=                0        DY3=                0
*/*/*/*/●/●/●/*/*/*/*/●/*/*/*/*/*/*/*/*/*/●/*/*/*/*/●/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/



V0=  3.400000000E 02        V1=  3.395000000E 02        V2=  3.380000000E 02        V3=  3.430000000E 02
Y0=                0        Y1=  1.000000000E 02        Y2=  2.500000000E 02        Y3=  5.000000000E 02

          MU1=  -5.0000000000E-03    MU2=  -1.0000000000E-02    MU3=   2.0000000000E-02
          D2=    5.0000000000E-01    K1=    9.9852941176E-01    DELT1= 2.9390138225E-03
          D3=    1.2500000000E 00    K2=    9.9411764706E-01    DELT2= 1.1730103783E-02
          GAMMA= 2.9911764706E 00    BETA=  7.4944812362E-01


                         **FOCUS FROM SECOND LAYER**

                             **** NO FOCUS ****


                         **FOCUS FROM THIRD LAYER**
                               SUBDOMAIN B
                 *ITERATION FOR EQUATION 600=  1.3998230875E 00
                 *ITERATION FOR EQUATION 600=  1.3784360286E 00
                 *ITERATION FOR EQUATION 600=  1.3725819734E 00
                 *ITERATION FOR EQUATION 600=  1.3709610508E 00
                 *ITERATION FOR EQUATION 600=  1.3705108164E 00
                 *ITERATION FOR EQUATION 600=  1.3703856480E 00
                 *ITERATION FOR EQUATION 600=  1.3703508418E 00
                 *ITERATION FOR EQUATION 600=  1.3703411625E 00
                 *ITERATION FOR EQUATION 600=  1.3703384706E 00
                 *ITERATION FOR EQUATION 600=  1.3703377220E 00
                 *ITERATION FOR EQUATION 600=  1.3703375138E 00
                 *ITERATION FOR EQUATION 600=  1.3703374559E 00
                 *ITERATION FOR EQUATION 600=  1.3703374398E 00
                 *ITERATION FOR EQUATION 600=  1.3703374353E 00
                 *ITERATION FOR EQUATION 600=  1.3703374341E 00

              FOCAL CONDITION=  8.7311491370E-11
              Q=      7.4835201688E 01    CTST=  8.6507341703E 00
              THOS=   6.5939602975E 00    THETM= 7.5834743998E 00
              XF=     9.1184258771E 03    SCNDD= 4.0770965498E 05
              OMEGA=  9.3076938468E-09    AIL=   7.7389463162E-01
```

There are two headings under which information is printed concerning the results of testing the atmosphere against the foci existence criteria; "FOCUS FROM SECOND LAYER" (Table 1, Line 28) and "FOCUS FROM THIRD LAYER" (Table 1, Line 33). If the existence conditions are not satisfied in either or both layers, the message "NO FOCUS" is printed (cf. Table 1, Line 30). If, however, the existence conditions are satisfied, meaning that a focus does exist, the focal distance, along with other pertinent information, is printed. On the occasions when there are two foci resulting from the third layer, the information is printed out in two consecutive sets. In the example (Table 1) there is no focus from the second layer and only one from the third.

There are nine quantities, all related to the solution of equation (1) or to x_F, which are printed out each time the program determines that there is a focus, whether resulting from atmospheric conditions in the second or third layer. These nine quantities are discussed in detail as follows.

1. FOCAL CONDITION (Table 1, Line 51) - This quantity is a measure of how accurately q has been determined, and should be very close to zero, since it is an evaluation of equation (1), using the value of q which has been obtained. In the example, FOCAL CONDITION $\approx 10^{-11}$, which leads one to believe q is quite accurate.

2. Q (Table 1, Line 52) - This is the solution of equation (1), no matter how obtained.

3. CTST (Table 1, Line 52) - This quantity is cotangent θ_o^*, where θ_o^* is the departure angle, measured from the horizontal, of the focal ray, when leaving the sound source.

4. THOS (Table 1, Line 53) - This is θ_o^*, having dimension degrees.

5. THETM (Table 1, Line 53) - This quantity is $\theta_{o,maximum}$, which is the largest angle of departure for rays which return to the sound source horizontal, y_o, from the third layer.* It is necessary that THETM be computed and printed out because of the mathematical treatment of equation (1). All the iteration procedures of Reference 2 are derived under the assumption that the third layer is infinitely extended, although, in practice, this is not the case. Hence, one must check the value (THOS - THETM). If this difference is negative, the focal ray is returned from the third layer. A positive difference means that the focal ray traverses the third layer entirely and enters the fourth, the third layer being too thin in such a case; the presumptive focal ray does not return from it and

* If the focus results from the second layer atmospheric conditions, THETM is the maximum angle of departure for rays which return to y_o from the second layer.

focus formation is precluded. In the example, (THOS - THETM) < 0; the third layer is extended far enough to enable the focal ray to turn around and return to the sound source horizontal where it gives rise to a focus.

6. XF (Table 1, Line 54) - This quantity is the focal distance, counted from the sound source. It has dimension meters if $[V]$ = m/sec, $[y]$ = meters; or feet if $[V]$ = ft/sec, $[y]$ = ft.

7. SCNDD (Table 1, Line 54) - This quantity is the second derivative of x_S with respect to θ_o, evaluated at $\theta_o = \theta_o^*$ and is printed out as a check.* It is used later in the computation of OMEGA and AIL.

8. OMEGA (Table 1, Line 55) - This is a quantity which is indicative of how strongly the focus intensity becomes infinite, and is proportional to the intensity average in a differential vicinity of a focus.

9. AIL (Table 1, Line 55) - This quantity is ($|\text{OMEGA}| \cdot x_F^2$). The average intensity in the vicinity of a focal point is found by $\bar{I}$ = AIL/ϵ where ϵ is an angular difference, taken at the sound source, between the focal ray and its differential neighbor. This average intensity can then be compared with some reference intensity, for instance, the average intensity associated with a homogeneous medium. The choice of the magnitude of ϵ is left to the discretion of the user, the only stipulation being that it should be chosen very small (on the order of 10^{-3} or less). This is because the expression for ω has been derived for $\epsilon \to 0$. Also, the intensity decreases very rapidly if one moves from a focus; one should stay very close to the focus to obtain a meaningful average. Therefore, ϵ should be chosen small enough in order for I to be larger than the reference value. If this reference value is taken as the average intensity I* near a standard focus, one usually chooses the same value of ϵ for the two foci, so that ϵ cancels when forming the intensity level $10 \log_{10} I/I^*$.

With this brief explanation of what the computer generates, the remainder of this report will deal with two problems which can be handled quite easily.

III. OBTAINING A FOCUS AT A DESIRED LOCATION

One of the problems mentioned earlier is that of varying the velocities and/or heights of a given velocity polygon in various manners calculated to result in focus formation at a particular location. The only way this can be handled is to run a number of closely related polygons through a computer

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* Since $dx_S/d\theta_o = 0$ at a focal point, SCNDD is identically equal to the curvature at x_F.

program for calculating sound ray landing distances, examine these landing distances and see if any of them are the desired focus. If not, more variations must be made and more polygons run. The time required to do this and the amount of computer output to be examined are dependent on how "skillful" the person running the program is in making the variations. There is no guarantee that any polygon can be found which yields a focus at a desired location.

The author of this report made such a study (cf. Ref. 3) in an attempt to obtain a focus at a distance of 27.2 km, by varying the parameters in an eleven-layered atmosphere. Observation suggested the presence of this focus, but it could not be obtained from the original velocity profile. This study took two weeks and an examination of fifty-six different sets of sound ray landing distances before one was found to yield a focus at 27.2 km.

Since this time, the program for determining the existence and location of focal points resulting from atmospheric conditions in a three-layered medium has been developed. It was decided to make the same study, but now replacing the eleven-layered atmosphere by one of only three layers and varying the parameters within the measurement accuracies.

Figure 1, solid line, is a plot of the original eleven-layered atmosphere, with the dashed line being its initial three-layer approximant. Table 2 gives a summary of the input parameters to the focal point determination program. The initial velocities and heights are given in columns 1 and 2, respectively, column 3 gives the increments of the velocities with column 4 denoting the number of times each is to be applied. This table shows that only two of the possible eight parameters were varied.

TABLE II

VI (m/sec)	YI (m)	ΔVI (m/sec)	CVI
339.98	0	0.0	1
345.54	721	0.0	1
345.25	858	0.0125	50
345.89	1016	0.0125	24

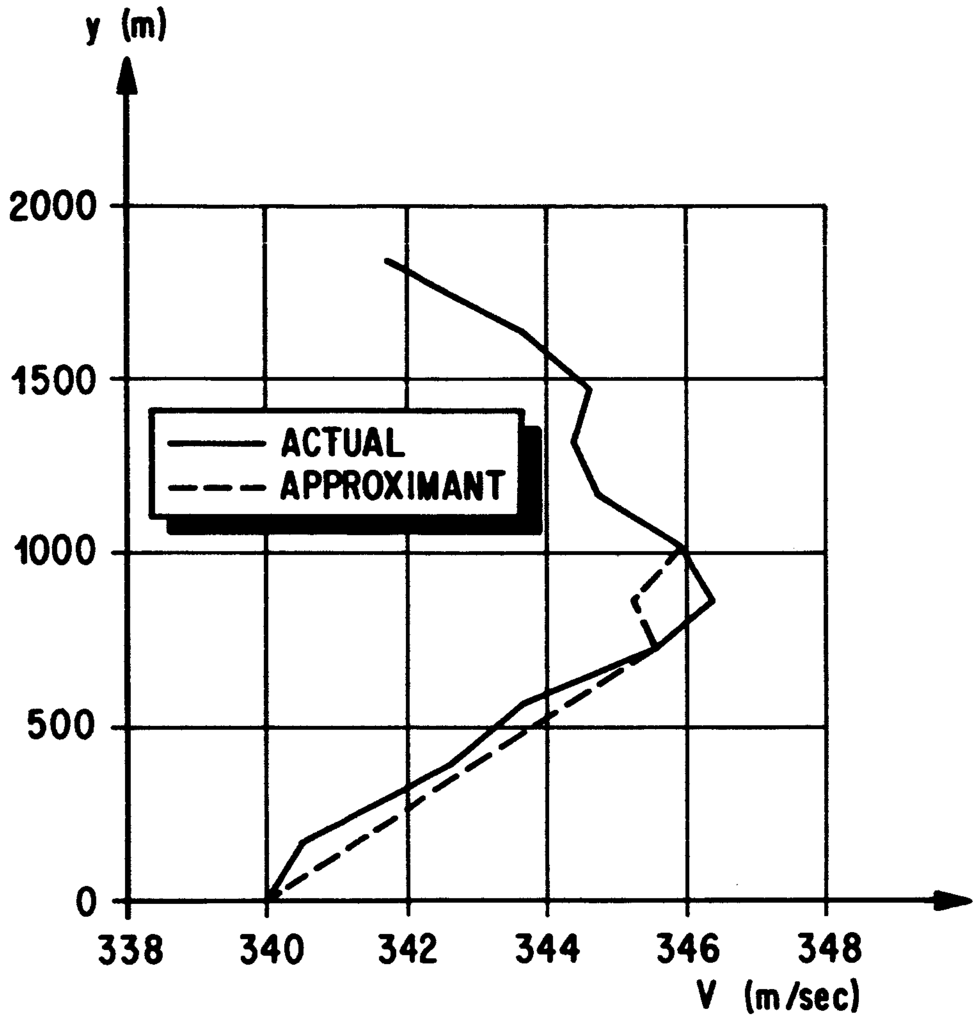

FIG. 1. VELOCITY OF SOUND POLYGON
ACTUAL VERSUS THREE LAYER APPROXIMANT

These increments resulted in a total of twelve hundred different profiles which were tested one by one for possible focus formation. The total computer running time was 28 minutes, which is considerably less than the more than three hours of running time the original study required, even though nearly twenty-two times as many profiles were examined. Moreover, the three-plus hours does not include the time required to check each profile for the focus, whereas the 28-minutes does. The study in Reference 3 did not include any estimate of the average intensity level in the vicinity of the focus, whereas the three-layered atmosphere program includes a measure of the average intensity level in the neighborhood of a focus. (See Appendix II for the derivation of this average intensity level.)

The results of this new study are summarized in Figures 2 and 3. Figure 2 gives the focal distance, x_F, plotted parametrically as a function of V_2 and V_3. Immediately one sees that if he is searching for a focus at 27.2 km, he has a whole host of (V_2, V_3) combinations from which to choose. Since the computer program calculates an ω for each x_F, the average intensity level in the vicinity of each x_F can be obtained. These results are given graphically in Figure 3, where the intensity level is plotted parametrically as a function of V_2 and V_3.* (The average intensity levels were computed under the assumption of D = 210 db and $\epsilon = 10^{-4}$; cf. Appendix II.)

As an example, let V_2 = 345.5 m/sec. Figure 2 shows that x_F = 27.2 km crosses V_2 = 345.5 m/sec at $V_3 \approx 346.09$ m/sec. For the same V_2 and the resulting V_3, Figure 3 indicates an average intensity level of 122 db. This same Figure 2 can be used to determine whether or not a particular (V_2, V_3) combination would yield a focus. When a focus does result for any (V_2, V_3) combination, Figure 3 will indicate the average intensity level about this focus.

IV. VARIATION OF X_F RESULTING FROM VARIATIONS IN V_0 AND V_3

As another example of the versatility of the focal point computation program, the reader's attention is called to Table III.

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* In both Figures 2 and 3, each continuous curve represents a constant value of V_3, with V_2 being the abscissa.

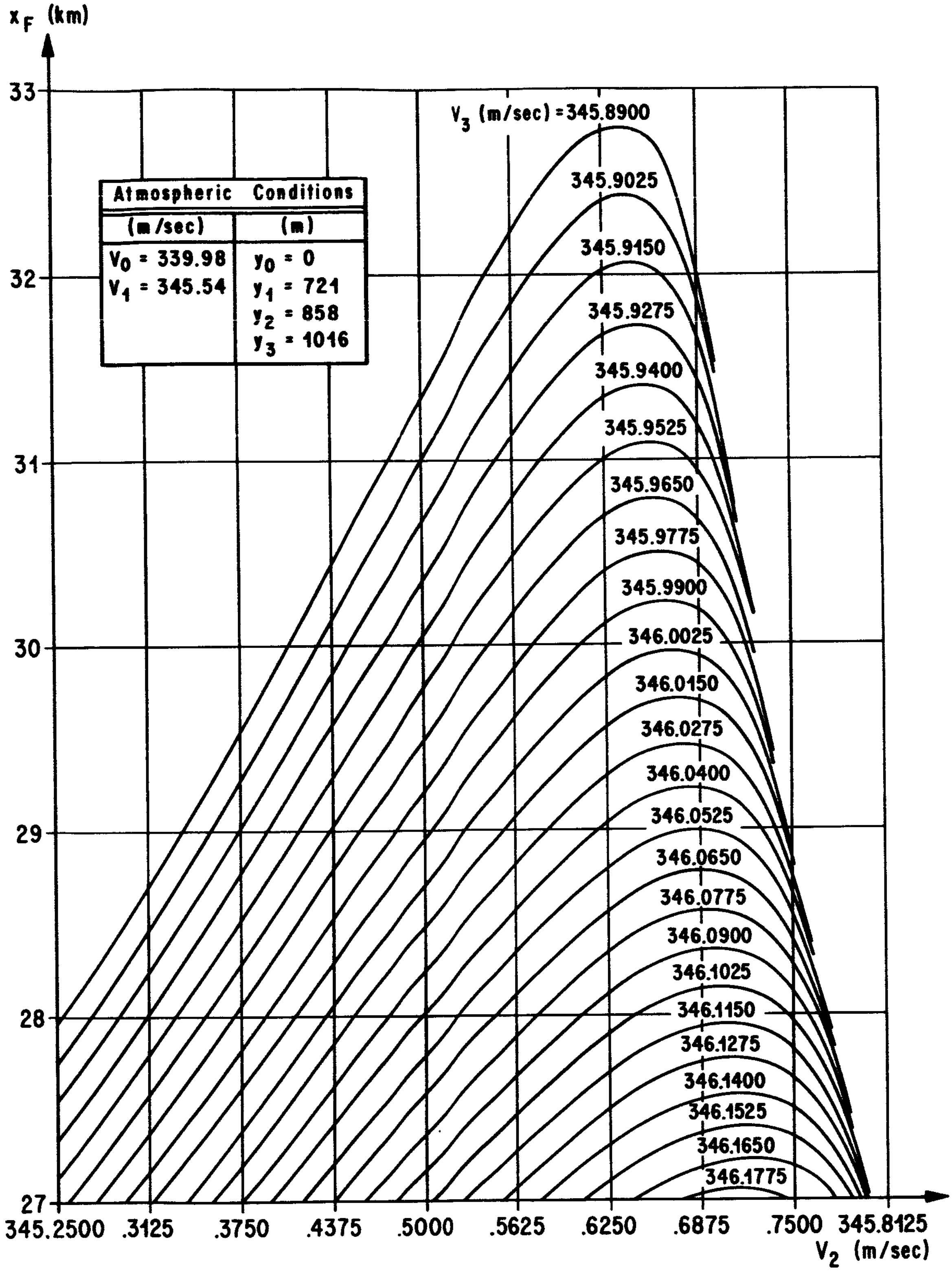

FIG. 2. VARIATION IN x_F RESULTING FROM VARIATIONS IN V_2 AND V_3

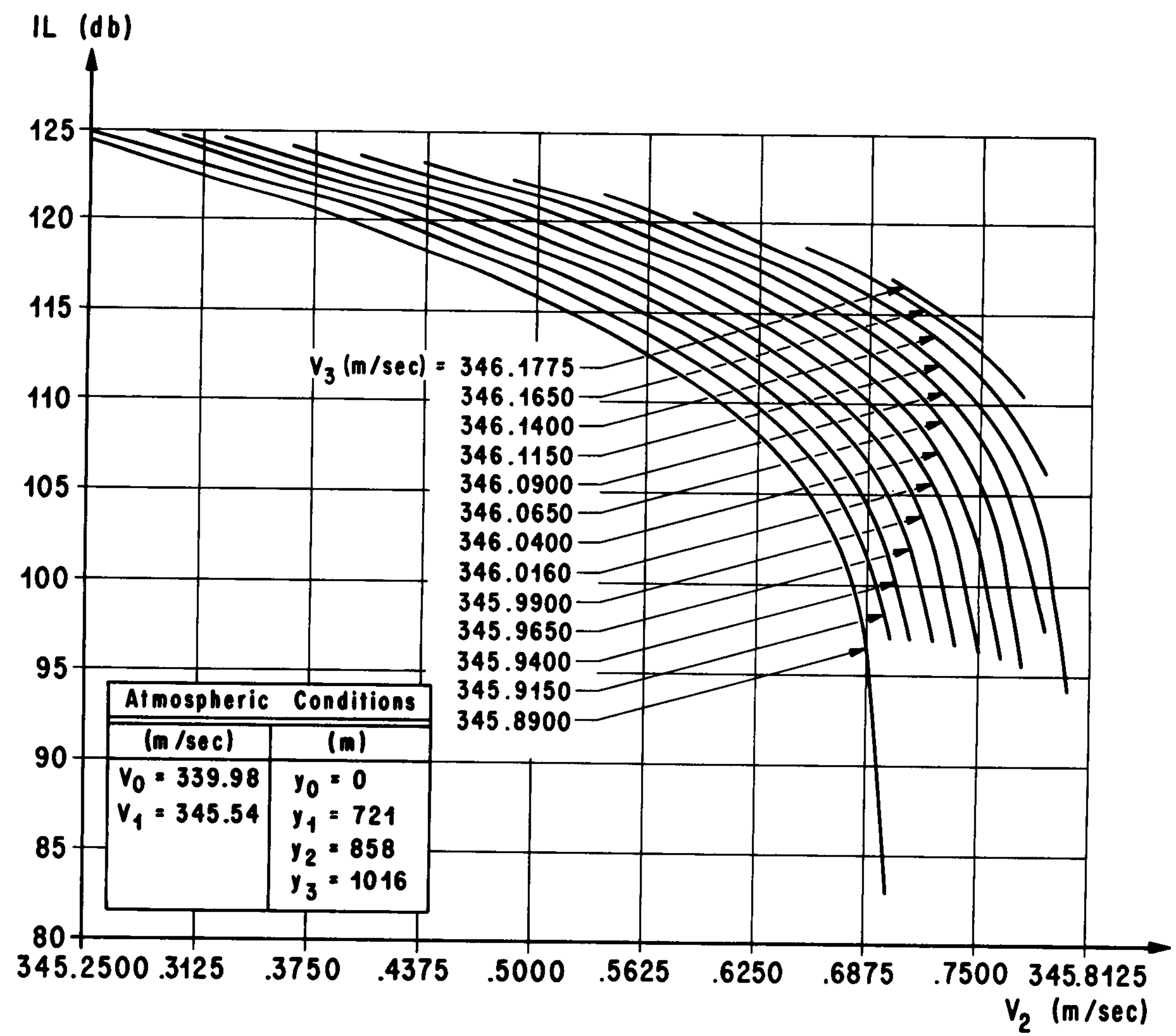

FIG. 3. VARIATION IN AVERAGE INTENSITY LEVEL RESULTING FROM VARIATIONS IN V_2 AND V_3

TABLE III

Input Parameters for Survey

$V_{initial}$ (m/sec)	$Y_{initial}$ (m)	ΔV (m/sec)	ΔY (m)	CVI	CYI
335.00	0.0	0.0625	0.0	21	1
336.00	100.0	0.0	0.0	1	1
336.25	125.0	0.0	0.0	1	1
336.25	170.0	0.0625	0.0	37	1
Tolerance 1 = 1.0×10^{-8}					

This table represents the input to the program for the purpose of making a survey of what happens to the focal distance, x_F, if V_0 and V_3 are allowed to vary simultaneously. These particular values were chosen because various combinations result in zero, one or two focal points. Seven hundred and seventy-seven different (V_0, V_3) combinations were examined, the results being summarized in Table IV.* A glance at this table shows immediately which (V_0, V_3) combinations yield no focal points, which combinations yield one focal point, and, perhaps most significantly, which combinations yield two focal points. Table IV also serves to stress a point which cannot be overemphasized; that is, that small changes in velocities may make significant changes not only in focus location but also in their very existence. For illustration, examine V_3 = 336.5 m/sec. For 335.0 m/sec $\leq V_0 \leq$ 336.125 m/sec, there is no focus formation, and for 336.25 m/sec $\leq V_0 \leq$ 336.4375 m/sec, there is only one focus formed. But for V_0 = 336.1875 m/sec there are two foci formed by rays returning from the third layer. Hence, for a very limited range of V_0, i.e., from V_0 = 336.125 to V_0 = 336.25, three different focus formation situations arise. The conclusion must be that V_0, in this region, has to be known quite accurately.

To further illustrate the amount of information which can be gained from proper use of the computer program, two additional figures are included here: Figure 4, which is x_F plotted parametrically as a function

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* 0 implies no focal point, 1 implies one focal point, 2 implies two focal points.

TABLE IV

V_3 \ V_o	335.0	335.0625	335.125	335.1875	335.25	335.3125	335.375	335.4375	335.5	335.5625	335.625	335.6875	335.75	335.8125	335.875	335.9375	336.0	336.0625	336.125	336.1875	336.25	336.3125	336.375	336.4375	336.5	336.5625	336.625	336.6875	336.75	336.8125	336.875	336.9375	337.0	337.0625	337.125	337.1875	337.25
336.25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
336.3125	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
336.375	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
336.4375	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
336.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
336.5625	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
336.625	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
336.6875	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
336.75	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
336.8125	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
336.875	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
336.9375	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
337.0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
337.0625	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
337.125	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
337.1875	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
337.25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
337.3125	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
337.375	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
337.4375	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
337.5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

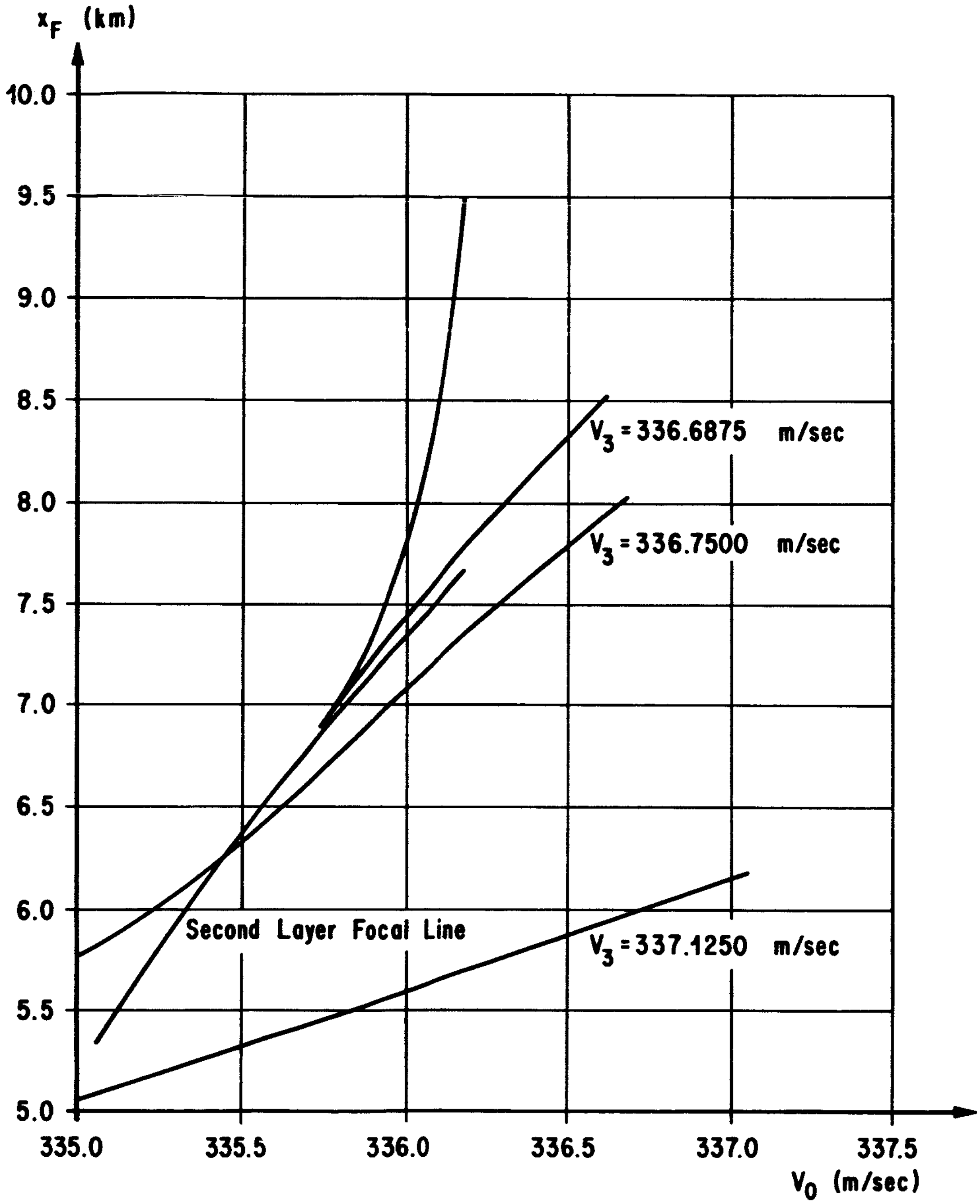

FIG. 4. x_F VERSUS V_0, WITH V_3 CONSTANT

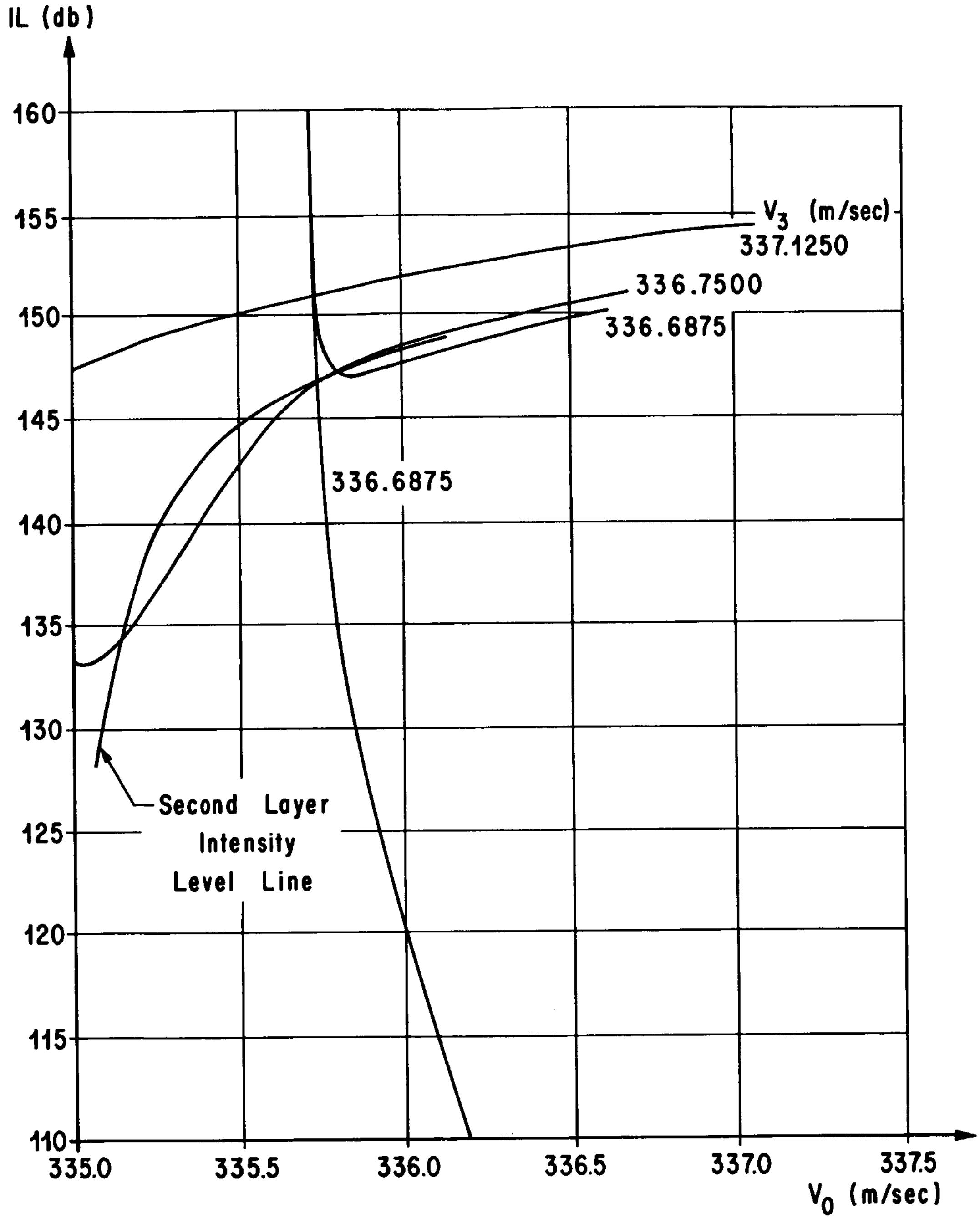

FIG. 5. INTENSITY LEVEL VERSUS V_0, WITH V_3 CONSTANT

of V_o and V_3 and Figure 5, which is the average intensity level plotted as a function of the same arguments, V_o and V_3.* Only a few of the 37 different V_3 curves are shown.

A close examination of Figure 4 reveals several interesting features, not the least of which is double-focus formation from the third layer, cf. the two-branched curve V_3 = 336.6875 m/sec. Note, however, that for V_o large enough, in this case $V_o > \approx 336.1825$ m/sec, only one focus is formed. Further, for V_o small enough, $V_o < \approx 335.25$ m/sec, no focus whatsoever is formed. As one would expect from an examination of the two-branched curve in Figure 4, there is a value of V_o ($\approx$ 335.75) for which the two focal points converge. This focal point convergence leads to inordinately high intensity levels, as the V_3 = 336.6875 m/sec curve in Figure 5 indicates. It has been shown analytically that the two branches actually converge to an infinite value of the average intensity level, so that the focus is uncommonly strong.

Furthermore, the curve labeled "second layer focal line" in Figure 4 denotes the focal points obtained as a result of atmospheric conditions in the second layer. They approach within 200 meters the cusp focus of the two-branched V_3 = 336.6875 m/sec curve. For this particular set of atmospheric conditions, the neighborhood around 6880 meters appears as one of very high intensity level. An examination of the corresponding average intensity level curves of Figure 5 verifies this conclusion. (When determining the average intensity level for a particular V_o value, the upper branch of x_F, when V_3 = 336.6875 m/sec, is associated with the lower branch of the average intensity level curve.)

Another undesirable situation is typified by the V_3 = 336.75 m/sec curve in Figure 4. This particular V_3-value permits only one focal point to develop from the third layer. However, the second layer focal line crosses this curve at $V_o \approx 335.4375$ m/sec. Here is a case of identical x_F's occurring from two different layers, which should lead to very high average intensity levels.

CONCLUSIONS

The computer program for direct computation of two- and three-layer focal points facilitates and expedites, to a significant degree, the systematic study of the relationship between atmospheric parameters and ground level acoustic focus location. It cannot answer all questions, neither does it solve all problems concerning focal points; but it does provide a valuable tool for further investigations both in the theoretical field and in the area of testing a given meteorological situation for possible focus formation.

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* The average intensity level is computed under the assumptions D = 210 db and $\epsilon = 1.0 \times 10^{-4}$.

APPENDIX I

```
      PROGRAM MABRY
 1000 READ(60,1)V0I,V1I,V2I,V3I,Y0I,Y1I,Y2I,Y3I
      READ(60,1200)DV0,DV1,DV2,DV3,DY0,DY1,DY2,DY3
 1200 FORMAT(8F9.0)
    1 FORMAT(4F15.0/4F15.0)
      READ(60,6)N0,N1,N2,N3,N4,N5,N6,N7
    6 FORMAT(8I5)
      READ(60,2)TOL1,LAST
    2 FORMAT(F10.0,I5)
      WRITE(61,3)TOL1
    3 FORMAT(1H1,49X,33HTHREE LAYERED ATMOSPHERE ANALYSIS///54X,13HTOLER
     1ANCE1=  F15.8)
      WRITE(61,40)N0,N1,N2,N3,N4,N5,N6,N7
   40 FORMAT(//12X,5HCV0= I3,6X,5HCV1= I3,6X,5HCV2= I3,6X,5HCV3= I3,6X,5
     1HCY0= I3,6X,5HCY1= I3,6X,5HCY2= I3,6X,5HCY3= I3)
      WRITE(61,41)V0I,V1I,V2I,V3I,Y0I,Y1I,Y2I,Y3I
   41 FORMAT(/10X,5HV0I= F15.10,10X,5HV1I= F15.10,10X,5HV2I= F15.10,10X,
     15HV3I= F15.10/10X,5HY0I= F15.10,10X,5HY1I= F15.10,10X,5HY2I= F15.1
     20,10X,5HY3I= F15.10)
      WRITE(61,42)DV0,DV1,DV2,DV3,DY0,DY1,DY2,DY3
   42 FORMAT(/10X,5HDV0= F15.10,10X,5HDV1= F15.10,10X,5HDV2= F15.10,10X,
     15HDV3= F15.10/10X,5HDY0= F15.10,10X,5HDY1= F15.10,10X,5HDY2= F15.1
     20,10X,5HDY3= F15.10)
      V0=V0I
      DO 30 I0=1,N0
      V1=V1I
      DO 31 I1=1,N1
      V2=V2I
      DO 32 I2=1,N2
      V3=V3I
      DO 33 I3=1,N3
      Y0=Y0I
      DO 34 I4=1,N4
      Y1=Y1I
      DO 35 I5=1,N5
      Y2=Y2I
      DO 36 I6=1,N6
      Y3=Y3I
      DO 37 I7=1,N7
      WRITE(61,6700)
 6700 FORMAT(120H            */*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*
     1/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/*/)
      WRITE(61,4)V0,V1,V2,V3,Y0,Y1,Y2,Y3
    4 FORMAT(////10X,5HV0=  F15.10,10X,5HV1=  F15.10,10X,5HV2=  F15.10,1
     10X,5HV3=  F15.10/10X,5HY0=  F15.10,10X,5HY1=  F15.10,10X,5HY2=  F1
     25.10,10X,5HY3=  F15.10)
      SMU1=(V1-V0)/Y1
      SMU2=(V2-V1)/(Y2-Y1)
      SMU3=(V3-V2)/(Y3-Y2)
      AK1=V1/V0
      AK2=V2/V0
      IF(ABS(V2-V1)-1.0E-09)4112,4112,4111
 4112 D2=9.999999999E206
      GO TO 4113
 4111 D2=1.0-SMU1/SMU2
```

```
4113 IF(ABS(V3-V2)-1.0E-09)4114,4114,4115
4114 D3=9.999999999E206
     GO TO 4116
4115 D3=1.0-SMU1/SMU3
4116 GA1=1.0-AK1**2
     GA2=1.0-AK2**2
     BETA=(AK1/(AK1+1.0))*((Y2-Y1)/Y1)
     GAMA=(AK2+1.0)*((Y2-Y1)/Y1)
     CALL SSWTCH(2,J)
     GO TO (4901,4902),J
4901 WRITE(61,7)SMU1,SMU2,SMU3
   7 FORMAT(/25X,6HMU1=  F17.10,5X,6HMU2=  F17.10,5X,6HMU3=  F17.10)
     WRITE(61,28)D2,AK1,GA1,D3,AK2,GA2,GAMA,BETA
  28 FORMAT(25X,6HD2=    F17.10,5X,6HK1=    F17.10,5X,6HDFLT1=F17.10/25X,
    16HD3=    F17.10,5X,6HK2=    F17.10,5X,6HDFLT2=F17.10/25X,6HGAMMA=F17
    2.10,5X,6HBETA= E17.10/)
4902 IF(ABS(SMU1-SMU2)-1.0E-07)9,9,8
   8 IF(ABS(SMU2-SMU3)-1.0E-07)11,11,6005
6005 WRITE(61,6006)
6006 FORMAT(/47X,27H**FOCUS FROM SECOND LAYER**)
     COMP=12.0
     GO TO 6010
   9 IF(SMU1)13,1411,15
1411 WRITE(61,1412)
1412 FORMAT(/47X,27H**FOCUS FROM SECOND LAYER**)
     WRITE(61,1413)
1413 FORMAT(/52X,18H**** NO FOCUS ****)
     GO TO 14
  14 IF(SMU3)1611,1611,17
1611 WRITE(61,1612)
1612 FORMAT(/47X,26H**FOCUS FROM THIRD LAYER**)
     WRITE(61,1613)
1613 FORMAT(/52X,18H**** NO FOCUS ****)
     GO TO 16
  17 WRITE(61,1612)
     CN1=V0/(SMU3*Y2)
     COTGTS=SQRT(CN1)
     WRITE(61,173)CN1,COTGTS
 173 FORMAT(33X,6HQ=     E17.10,5X,6HCTST= E17.10)
     XF=4.*Y2*COTGTS
     CN2=V0/(SMU3*Y2+V0)
     THETS=ACOS(SQRT(CN2))
     THETM=(ACOS(V0/V3))*57.29577951
     SCNDD=(8.*Y2*(1.+CN1)**2)*COTGTS
     OMEGE=(4.*COTGTS)/(XF*SCNDD)
     THOS=THETS*57.29577951
     AIL=ABS(OMEGE)*XF*XF
     WRITE(61,2078)THOS,THETM,XF,SCNDD,OMEGE,AIL
2078 FORMAT(33X,6HTHOS= F17.10,5X,6HTHETM=F17.10/33X,6HXF=    F17.10,5X,
    16HSCNDD=F17.10/33X,6HOMEGA=E17.10,5X,6HAIL=  E17.10//)
     GO TO 16
  13 IF(SMU3)1666,1666,19
1666 WRITE(61,1412)
     WRITE(61,1413)
     WRITE(61,1612)
```

```
      WRITE(61,1613)
      GO TO 16
   15 IF(SMU3-SMU1)1666,1666,19
   19 CN4=((V2-V0)*(Y3-Y2))/((V3-V2)*Y2)
      CN3=2.*CN4
      CN5=CN4**2
      CN6=(V2/V0)**2
      CN7=(1.0-CN4)**2
      CN8=((CN3-CN5)/(CN6-CN7))
      WRITE(61,1412)
      WRITE(61,1413)
      WRITE(61,1612)
      Q=CN8/(1.-CN8)
      FC=1.-D3/SQRT(1.+GA2*Q)
      WRITE(61,241)FC
      CTST=SQRT(Q)
      WRITE(61,173)Q,CTST
      COSTHS=SQRT(CN8)
      THETS=ACOS(COSTHS)
      THETM=(ACOS(V0/V2))*57.29577951
      CN9=((V2+V0)/(V0-V2))
      CN10=CN9*(CN7-1.0)
      CN11=SQRT(CN10)
      XF=2.*Y2*CN11
      SCNDD=(2.0*V0*(1.-AK2**2)*(1./SMU3-1./SMU1))/(COSTHS*(1.-AK2**2*CN
     18)**1.5)
      OMEGE=(4.*(COSTHS/SIN(THETS)))/(XF*SCNDD)
      THOS=THETS*57.29577951
      AIL=ABS(OMEGE)*XF*XF
      WRITE(61,2050)THOS,THETM,XF,SCNDD,OMEGE,AIL
 2050 FORMAT(33X,6HTHOS= E17.10,5X,6HTHETM=E17.10/33X,6HXF=    E17.10,5X,
     16HSCNDD=E17.10/33X,6HOMEGA=E17.10,5X,6HAIL=   E17.10//)
      GO TO 16
   11 COMP=35.0
      IF(V3-V0)6010,6010,4513
 4513 THETM=(ACOS(V0/V3))*57.29577951
 6010 IF(SMU1)20,21,22
   21 IF(SMU2)1030,1030,23
 1030 WRITE(61,1613)
      GO TO 10
   23 CN12=V0/(SMU2*Y1)
      FC=CN12-(Y2-Y1)/(Y1*(AK2-1.))
      WRITE(61,241)FC
      COTGTS=SQRT(CN12)
      WRITE(61,173)CN12,COTGTS
      XF=4.*Y1*COTGTS
      THETS=ATAN(1.0/COTGTS)
      THETM=(ACOS(V0/V2))*57.29577951
      SCNDD=(4.*Y1*(1.+CN12)**2)/COTGTS
      OMEGE=(4.*COTGTS)/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      GO TO 6009
   20 IF(SMU2)1020,1020,2009
 1020 WRITE(61,1613)
      GO TO 10
```

```
2009 IF(V2-V0)101,101,24
 101 WRITE(61,1613)
     GO TO 10
  22 IF(SMU2-SMU1)1010,1010,24
1010 WRITE(61,1613)
     GO TO 10
  24 CN13=1.0-D2**2
     CN14=AK1**2-D2**2
     CN15=SQRT(CN13/CN14)
     Q=CN15/SQRT(1.-CN15**2)
     CTST=SQRT(Q)
     FC=1.-(D2*SQRT(1.-CN15**2))/SQRT(1.-(AK1*CN15)**2)
     WRITE(61,241)FC
 241 FORMAT(/33X,17HFOCAL CONDITION= E17.10)
     WRITE(61,173)Q,CTST
     THETS=ACOS(CN15)
     THETM=(ACOS(V0/V2))*57.29577951
     CN16=1.0+AK1
     CN17=1.0-AK1
     CN18=D2**2-1.0
     CN19=(CN16/CN17)*CN18
     CN20=SQRT(CN19)
     XF=2.*Y1*CN20
     SCNDD=(2.*V0*(1./SMU2-1.0/SMU1)*(1.-AK1**2))/(CN15*(1.-AK1**2*CN15
    1**2)**1.5)
     COTGTS=CN15/SQRT(1.-CN15**2)
     OMEGE=(4.*COTGTS)/(XF*SCNDD)
     AIL=ABS(OMEGE)*XF*XF
6009 THOS=THETS*57.29577951
     WRITE(61,25)THOS,THETM,XF,SCNDD,OMEGE,AIL
     IF(COMP-12.0)16,10,16
  18 THOS=THETS*57.29577951
     WRITE(61,25)THOS,THETM,XF,SCNDD,OMEGE,AIL
  25 FORMAT(33X,6HTHOS= E17.10,5X,6HTHETM=E17.10/33X,6HXF=   E17.10,5X,
    16HSCNDD=E17.10/33X,6HOMEGA=E17.10,5X,6HAIL=  E17.10//)
     GO TO 16
  10 IF(SMU3)1611,1611,1276
1276 IF(V3-V0)1611,1611,12
  12 WRITE(61,27)
  27 FORMAT(//47X,26H**FOCUS FROM THIRD LAYER**)
     IF(SMU3-SMU2)29,29,1031
1031 IF(SMU2)38,1100,38
1100 D2=9.999999999E206
  38 IF(D2)39,43,44
  43 WRITE(61,45)SMU1,SMU2
  45 FORMAT(/9X,29HTWO LAYER CASE WHERE MU1=MU2 E14.5,10X,E14.5)
     GO TO 16
  39 WRITE(61,6245)
6245 FORMAT(57X,11HSUBDOMAIN A)
     CN21=1.0-GA1/GA2
     CN22=D2/(D2-D3)
     S1=(1.0/CN21)*CN22
     CN23=-1.0/(D2-D3)
     CN24=GA1/GA2
  48 CN25=(1.0-CN24)*S1**2+CN24
```

```
      CN26=SQRT(CN25)
      S2=CN23+CN22*(S1/CN26)
      CALL SSWTCH(1,J)
      GO TO (6200,6201),J
 6200 WRITE(61,2191)S1
 2191 FORMAT(40X,29H*ITERATION FOR EQUATION 500= E17.10)
 6201 CN27=ABS(S1-S2)
      IF(CN27-TOL1)46,46,47
   47 S1=S2
      GO TO 48
   46 Q=(1.0/GA2)*((1.0/S2**2)-1.0)
      FC=1.-((D2)/SQRT(1.+GA1*Q))+((D2-D3)/SQRT(1.+GA2*Q))
      WRITE(61,241)FC
      CTST=SQRT(Q)
      WRITE(61,173)Q,CTST
      CN28=1.0/CTST
      THETS=ATAN(CN28)
      THETM=(ACOS(V0/V3))*57.29577951
      CN29=SQRT(1.0+GA1*Q)
      CN30=SQRT(1.0+GA2*Q)
      CN31=D2-D3
      XF=((2.*Y1)/(AK1-1.0))*(1.0/CTST)*(1.0-D2*CN29+CN31*CN30)
      SCNDD=((2.*V0*(1.+Q)**2)/(SMU1*CTST))*((D2-D3)*GA2/(1.+GA2*Q)**1.5
     1-D2*GA1/(1.+GA1*Q)**1.5)
      OMEGE=4.*CTST/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      GO TO 18
   44 IF(D2-1.0)49,502,51
  502 IF(GAMA-1.0)52,53,54
   52 S1=1.0-GAMA*(SMU2/SMU3)
   55 S2=((1.0-GAMA)*S1**0.3333333333)+GAMA*(1.0-SMU2/SMU3)
      CALL SSWTCH(1,J)
      GO TO (6202,6203),J
 6202 WRITE(61,2091)S1
 2091 FORMAT(40X,29H*ITERATION FOR EQUATION 750= E17.10)
 6203 CN33=ABS(S1-S2)
      IF(CN33-TOL1)56,56,57
   57 S1=S2
      GO TO 55
   56 Q=(S2**0.6666666667-1.0)/GA2
      CTST=SQRT(Q)
      FC=1./SMU2+(1./SMU3-1./SMU2)/SQRT(1.+GA2*Q)-(Y1*Q)/V0
      WRITE(61,241)FC
      WRITE(61,173)Q,CTST
      CN34=1.0/CTST
      THETS=ATAN(CN34)
      THETM=(ACOS(V0/V3))*57.29577951
      CN35=SQRT(1.0+GA2*Q)
      XF=4.*Y1*(1.+((AK2+1.)/2.)*((Y2-Y1)/Y1)*(1.-SMU2/SMU3)/CN35)*CTST
      SCNDD=(2.*V0*(1.+Q)**2/CTST)*(GA2*(1./SMU3-1./SMU2)/(1.+GA2*Q)**1.
     15+2.*Y1/V0)
      OMEGE=4.*CTST/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      GO TO 18
   54 S1=1./((1.-SMU2/SMU3)**3)
```

```
   58 S2=(1.+(GAMA-1.)*S1**0.6666666667)/(GAMA*(1.-SMU2/SMU3))
      CALL SSWTCH(1,J)
      GO TO (6204,6205),J
 6204 WRITE(61,59)S1
   59 FORMAT(40X,29H*ITERATION FOR EQUATION 775= E17.10)
 6205 CN36=ABS(S1-S2)
      IF(CN36-TOL1)60,60,61
   61 S1=S2
      GO TO 58
   60 Q=(1./GA2)*((1./S2**0.6666666667)-1.)
      FC=1.-((D2)/SQRT(1.+GA1*Q))+((D2-D3)/SQRT(1.+GA2*Q))
      WRITE(61,241)FC
      CTST=SQRT(Q)
      WRITE(61,173)Q,CTST
      CN37=1./CTST
      THETS=ATAN(CN37)
      THETM=(ACOS(V0/V3))*57.29577951
      CN38=SQRT(1.+GA2*Q)
      XF=4.*Y1*(1.+((AK2+1.)/2.)*((Y2-Y1)/Y1)*(1.-SMU2/SMU3)/CN38)*CTST
      SCNDD=(2.*V0*(1.+Q)**2/CTST)*(GA2*(1./SMU3-1./SMU2)/(1.+GA2*Q)**1.
     15+2.*Y1/V0)
      OMEGE=4.*CTST/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      GO TO 18
   53 Q=(1.-SMU2/SMU3)**0.6666666667-1.0
      CTST=SQRT(Q)
      WRITE(61,173)Q,CTST
      CN39=1./CTST
      THETS=ATAN(CN39)
      THETM=(ACOS(V0/V3))*57.29577951
      CN40=SQRT(1.+GA2*Q)
      XF=4.*Y1*(1.+((AK2+1.)/2.)*((Y2-Y1)/Y1)*(1.-SMU2/SMU3)/CN40)*CTST
      SCNDD=(2.*V0*(1.+Q)**2/CTST)*(GA2*(1./SMU3-1./SMU2)/(1.+GA2*Q)**1.
     15+2.*Y1/V0)
      OMEGE=4.*CTST/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      GO TO 18
   49 WRITE(61,6246)
 6246 FORMAT(57X,11HSUBDOMAIN B)
      S1=1.+((D3-1.)/(1.-D2*(1.-GA1/GA2)))
   62 CN41=(1.-GA1/GA2)+(GA1/GA2)*S1**2
      CN42=SQRT(CN41)
      S2=D3-D2+D2*S1/CN42
      CALL SSWTCH(1,J)
      GO TO (6206,6207),J
 6206 WRITE(61,63)S1
   63 FORMAT(40X,29H*ITERATION FOR EQUATION 600= E17.10)
 6207 CN43=ABS(S1-S2)
      IF(CN43-TOL1)64,64,65
   65 S1=S2
      GO TO 62
   64 Q=(S2**2-1.)/GA2
      FC=1.-((D2)/SQRT(1.+GA1*Q))+((D2-D3)/SQRT(1.+GA2*Q))
      WRITE(61,241)FC
      CTST=SQRT(Q)
```

```
      WRITE(61,173)Q,CTST
      CN44=1./CTST
      THETS=ATAN(CN44)
      THETM=(ACOS(V0/V3))*57.29577951
      XF=(2.*Y1/(AK1-1.))*(1./CTST)*(1.-D2*SQRT(1.+GA1*Q)+(D2-D3)*SQRT(1
     1.+GA2*Q))
      SCNDD=((2.*V0*(1.+Q)**2)/(SMU1*CTST))*((D2-D3)*GA2/(1.+GA2*Q)**1.5
     1-D2*GA1/(1.+GA1*Q)**1.5)
      OMEGE=4.*CTST/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      GO TO 18
   51 IF(D2-Y2/Y1)66,67,68
   67 CN45=(1.-((Y2-Y1)/Y2)*(SMU2/SMU3))**2
      Q=(1./GA1)*(1./CN45-1.)
      FC=1.-(D2/SQRT(1.+GA1*Q))+((D2-D3)/SQRT(1.+GA2*Q))
      WRITE(61,241)FC
      CTST=SQRT(Q)
      WRITE(61,173)Q,CTST
      CN46=1./CTST
      THETS=ATAN(CN46)
      THETM=(ACOS(V0/V3))*57.29577951
      XF=2.*(AK1+1.)*Y2*SQRT(Q/(1.+GA1*Q))
      SCNDD=-(2.*V0*D2*GA1*(1.+Q)**2)/(SMU1*CTST*(1.+GA1*Q)**1.5)
      OMEGE=4.*CTST/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      GO TO 18
   66 WRITE(61,6247)
 6247 FORMAT(57X,11HSUBDOMAIN C)
      S1=1.0
      CN47=(1.-GA1/GA2)
   69 CN48=((D2/(S1+D2-D3))**2)-GA1/GA2
      S2=SQRT(CN47/CN48)
      CALL SSWTCH(1,J)
      GO TO (6208,6209),J
 6208 WRITE(61,70)S1
   70 FORMAT(40X,29H*ITERATION FOR EQUATION 800= E17.10)
 6209 CN49=ABS(S1-S2)
      IF(CN49-TOL1)71,71,72
   72 S1=S2
      GO TO 69
   71 Q=(S2**2-1.0)/GA2
      FC=1.-((D2)/SQRT(1.+GA1*Q))+((D2-D3)/SQRT(1.+GA2*Q))
      WRITE(61,241)FC
      CTST=SQRT(Q)
      WRITE(61,173)Q,CTST
      CN50=1./CTST
      THETS=ATAN(CN50)
      THETM=(ACOS(V0/V3))*57.29577951
      XF=(2.*Y1/(AK1-1.))*(1./CTST)*(1.-D2*SQRT(1.+GA1*Q)+(D2-D3)*SQRT(1
     1.+GA2*Q))
      SCNDD=((2.*V0*(1.+Q)**2)/(SMU1*CTST))*((D2-D3)*GA2/(1.+GA2*Q)**1.5
     1-D2*GA1/(1.+GA1*Q)**1.5)
      OMEGE=4.*CTST/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      GO TO 18
```

```
   68 IF(SMU2)73,7431,73
   73 WRITE(61,6248)
 6248 FORMAT(57X,11HSUBDOMAIN D)
      S1=(1./(1.-GA2/GA1))*((D2-D3)/D2)
   75 CN10=(1.-GA2/GA1)*S1**2+GA2/GA1
      S2=1./D2+((D2-D3)/D2)*(S1/SQRT(CN10))
      CALL SSWTCH(1,J)
      GO TO (6210,6211),J
 6210 WRITE(61,147)S1
  147 FORMAT(40X,30H*ITERATION FOR EQUATION 1000= E17.10)
 6211 IF(ABS(S1-S2)-TOL1)77,77,78
   78 S1=S2
      GO TO 75
   77 Q=(1./GA1)*(1./S2**2-1.)
      FC=1.-((D2)/SQRT(1.+GA1*Q))+((D2-D3)/SQRT(1.+GA2*Q))
      WRITE(61,241)FC
      CTST=SQRT(Q)
      WRITE(61,173)Q,CTST
      CN11=1./CTST
      THETS=ATAN(CN11)
      THETM=(ACOS(V0/V3))*57.29577951
      XF=(2.*Y1/(AK1-1))*(1./CTST)*(1.-D2*SQRT(1.+GA1*Q)+(D2-D3)*SQRT(1.
     1+GA2*Q))
      SCNDD=((2.*V0*(1.+Q)**2)/(SMU1*CTST))*((D2-D3)*GA2/(1.+GA2*Q)**1.5
     1-D2*GA1/(1.+GA1*Q)**1.5)
      OMEGE=4.*CTST/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      GO TO 18
 7431 IF(D3-BETA)79,80,81
   79 S1=(2./(3.*BETA))+((BETA-D3)/BETA)**1.5
   76 S2=(1.+(BETA-D3)*S1**0.3333333333)/BETA
      CALL SSWTCH(1,J)
      GO TO (6212,6213),J
 6212 WRITE(61,92)S1
   92 FORMAT(40X,30H*ITERATION FOR EQUATION 1175= E17.10)
 6213 IF(ABS(S1-S2)-TOL1)145,145,146
  146 S1=S2
      GO TO 76
  145 Q=(1./GA1)*(1./S2**0.6666666666-1.)
      FC=1.-D3/SQRT(1.+GA1*Q)+(Q*GA1*AK1*(Y2-Y1))/(Y1*(AK1+1.)*(1.+GA1*Q
     1)**1.5)
      WRITE(61,241)FC
      CTST=SQRT(Q)
      WRITE(61,173)Q,CTST
      CN12=1./CTST
      THETS=ATAN(CN12)
      THETM=(ACOS(V0/V3))*57.29577951
      XF=(2.*Y1/CTST)*((1.-D3*SQRT(1.+GA1*Q))/(AK1-1.)+((Y2-Y1)/Y1)*(AK1
     1*Q/SQRT(1.+GA1*Q)))
      SCNDD=(2.*V0*(1.+Q)**2/(SMU1*CTST))*((SMU1*(Y2-Y1)*(2.*AK1**2-AK1*
     1*2*GA1*Q))/(V1*(1.+GA1*Q)**2.5)-(D3*GA1/(1.+GA1*Q)**1.5))
      OMEGE=4.*CTST/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      GO TO 18
   80 Q=(BETA**0.6666666667-1.0)/GA1
```

```
      CTST=SQRT(Q)
      WRITE(61,173)Q,CTST
      CN13=1./CTST
      THETS=ATAN(CN13)
      THETM=(ACOS(V0/V3))*57.29577951
      XF=(2.*Y1/CTST)*((1.-D3*SQRT(1.+GA1*Q))/(AK1-1.0)+((Y2-Y1)/Y1)*(AK
     11*Q/SQRT(1.+GA1*Q)))
      SCNDD=(6.*Y1*(AK1+1.)*(1.+Q)**2)/(SQRT(Q)*(1.+GA1*Q))
      OMEGE=(4.*CTST)/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      GO TO 18
   81 S1=(D3-BETA)**3+3.*BETA
   82 S2=BETA+(D3-BETA)*S1**0.6666666667
      CALL SSWTCH(1,J)
      GO TO (6214,6215),J
 6214 WRITE(61,83)S1
   83 FORMAT(40X,30H*ITERATION FOR EQUATION 1125= E17.10)
 6215 IF(ABS(S1-S2)-TOL1)84,84,85
   85 S1=S2
      GO TO 82
   84 Q=(S2**0.66666666667-1.0)/GA1
      FC=1.-D3/SQRT(1.+GA1*Q)+(Q*GA1*AK1*(Y2-Y1))/(Y1*(AK1+1.)*(1.+GA1*Q
     1)**1.5)
      WRITE(61,241)FC
      CTST=SQRT(Q)
      WRITE(61,173)Q,CTST
      CN14=1.0/CTST
      THETS=ATAN(CN14)
      THETM=(ACOS(V0/V3))*57.29577951
      XF=(2.*Y1/CTST)*((1.-D3*SQRT(1.+GA1*Q))/(AK1-1.)+((Y2-Y1)/Y1)*(AK1
     1*Q/SQRT(1.+GA1*Q)))
      SCNDD=(2.*V0*(1.+Q)**2/(SMU1*CTST))*((SMU1*(Y2-Y1)*(2.*AK1**2-AK1*
     1*2*GA1*Q))/(V1*(1.+GA1*Q)**2.5)-(D3*GA1/(1.+GA1*Q)**1.5))
      OMEGE=4.*CTST/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      GO TO 18
   29 IF(D2)87,87,86
   87 WRITE(61,6249)
 6249 FORMAT(57X,11HSUBDOMAIN A)
      WRITE(61,1613)
      GO TO 16
   86 IF(D2-1.0)89,90,91
   90 CSTR=((1.-GAMA)/3.)**3-(GAMA/2.*(SMU2/SMU3-1.))**2
      WRITE(61,9088)CSTR
 9088 FORMAT(33X,6HCSTAR=E17.10)
      IF(CSTR)87,93,94
   93 Q=(((1.-GAMA)/3.)-1.)*1./GA2
      CTST=SQRT(Q)
      WRITE(61,173)Q,CTST
      CN15=1./CTST
      XF=4.*Y1*CTST*(1.+(AK2+1.)/2.*(Y2-Y1)/Y1*(1.-SMU2/SMU3)/SQRT(1.+GA
     12*Q))
      THETS=ATAN(CN15)
      THETM=(ACOS(V0/V3))*57.29577951
      SCNDD=(2.*V0*(1.+Q)**2/CTST)*(GA2*(1./SMU3-1./SMU2)/(1.+GA2*Q)**1.
```

```
     15+2.*Y1/V0)
      OMEGE=4.*CTST/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      GO TO 18
   94 J2P=-1
      S1=(GAMA/(GAMA-1.0))*(SMU2/SMU3-1.0)
   95 S2=(S1**3+GAMA*(SMU2/SMU3-1.0))/(1.0-GAMA)
      IF(ABS(S1-S2)-TOL1)96,96,97
   97 S1=S2
      CALL SSWTCH(1,J)
      GO TO (6216,6217),J
 6216 WRITE(61,98)S1
   98 FORMAT(40X,30H*ITERATION FOR EQUATION 1375= E17.10)
 6217 GO TO 95
   96 Q=(S2**2-1.0)/GA2
      FC=1.-((D2)/SQRT(1.+GA1*Q))+((D2-D3)/SQRT(1.+GA2*Q))
      WRITE(61,241)FC
      CTST=SQRT(Q)
      WRITE(61,173)Q,CTST
      CN25=1./CTST
      THETS=ATAN(CN25)
      THETM=(ACOS(V0/V3))*57.29577951
      XF=4.*Y1*(1.+((AK2+1.)/2.)*((Y2-Y1)/Y1)*(1.-SMU2/SMU3)/SQRT(1.+GA2
     1*Q))*(CTST)
      SCNDD=(2.*V0*(1.+Q)**2/CTST)*(GA2*(1./SMU3-1./SMU2)/(1.+GA2*Q)**1.
     15+2.*Y1/V0)
      OMEGE=4.*CTST/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      J1P=0
  100 THOS=THETS*57.29577951
      WRITE(61,25)THOS,THETM,XF,SCNDD,OMEGE,AIL
      IF(J1P)16,102,16
  102 IF(J2P)103,104,105
  103 J1P=4
      S1=(1.-GAMA)*(SMU2/SMU3)
  106 S2=(1.0-GAMA)*S1**0.3333333333+GAMA*(1.0-SMU2/SMU3)
      CALL SSWTCH(1,J)
      GO TO (6218,6219),J
 6218 WRITE(61,107)S1
  107 FORMAT(40X,30H*ITERATION FOR EQUATION 1376= E17.10)
 6219 IF(ABS(S1-S2)-TOL1)108,108,109
  109 S1=S2
      GO TO 106
  108 Q=(S2**0.6666666667-1.0)/GA2
      FC=1.-((D2)/SQRT(1.+GA1*Q))+((D2-D3)/SQRT(1.+GA2*Q))
      WRITE(61,241)FC
      CTST=SQRT(Q)
      WRITE(61,173)Q,CTST
      CN31=1./CTST
      THETS=ATAN(CN31)
      THETM=(ACOS(V0/V3))*57.29577951
      XF=4.*Y1*(1.+((AK2+1.)/2.)*((Y2-Y1)/Y1)*(1.-SMU2/SMU3)/SQRT(1.+GA2
     1*Q))*(CTST)
      SCNDD=(2.*V0*(1.+Q)**2/CTST)*(GA2*(1./SMU3-1./SMU2)/(1.+GA2*Q)**1.
     15+2.*Y1/V0)
```

```
      OMEGE=4.*CTST/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      GO TO 100
   89 WRITE(61,6251)
 6251 FORMAT(57X,11HSUBDOMAIN B)
      CCO1=(D2**2/(1.0-GA1/GA2))**0.3333333333
      CCO2=((D2-D3)**2/(GA2/GA1-1.0))**0.3333333333
      CE=CCO1-CCO2
      WRITE(61,8991)CE
 8991 FORMAT(/33X,6HCE=    E17.10)
      IF(CE-1.0)110,111,112
  110 WRITE(61,113)
  113 FORMAT(/52X,18H**** NO FOCUS ****)
      GO TO 16
  111 Q=((D2*(1.-GA1/GA2))**0.6666666667-1.0)/GA1
      CTST=SQRT(Q)
      WRITE(61,173)Q,CTST
      CN44=1./CTST
      THETS=ATAN(CN44)
      THETM=(ACOS(V0/V3))*57.29577951
      XF=(2.*Y1)/(AK1-1.0)*1./CTST*(1.-D2*SQRT(1.+GA1*Q)+(D2-D3)*SQRT(1.
     1+GA2*Q))
      SCNDD=((2.*V0*(1.+Q)**2)/(SMU1*CTST))*((D2-D3)*GA2/(1.+GA2*Q)**1.5
     1-D2*GA1/(1.+GA1*Q)**1.5)
      OMEGE=4.*CTST/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      GO TO 18
  112 J1P=0
      J2P=0
      S1=(D2/SQRT(1.-GA1/GA2)-1.)/(D2-D3)
  114 CN45=SQRT((1.-GA1/GA2)*S1**2+GA1/GA2)
      S2=(D2/(D2-D3))*S1/CN45-1./(D2-D3)
      CALL SSWTCH(1,J)
      GO TO (6220,6221),J
 6220 WRITE(61,115)S1
  115 FORMAT(40X,30H*ITERATION FOR EQUATION 1275= E17.10)
 6221 IF(ABS(S1-S2)-TOL1)116,116,117
  117 S1=S2
      GO TO 114
  116 Q=(1./GA2)*(1./S2**2-1.0)
      FC=1.-((D2)/SQRT(1.+GA1*Q))+((D2-D3)/SQRT(1.+GA2*Q))
      WRITE(61,241)FC
      CTST=SQRT(Q)
      WRITE(61,173)Q,CTST
      CN36=1./CTST
      THETS=ATAN(CN36)
      THETM=(ACOS(V0/V3))*57.29577951
      XF=(2.*Y1/(AK1-1.))*1./CTST*(1.-D2*SQRT(1.+GA1*Q)+(D2-D3)*SQRT(1.+
     1GA2*Q))
      SCNDD=((2.*V0*(1.+Q)**2)/(SMU1*CTST))*((D2-D3)*GA2/(1.+GA2*Q)**1.5
     1-D2*GA1/(1.+GA1*Q)**1.5)
      OMEGE=4.*CTST/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      GO TO 100
  104 J1P=4
```

```
      S1=1.-(1.-D3)/(1.-(D2-D3)*(GA2/GA1-1.))
  118 CN55=SQRT((1.-GA2/GA1)+GA2/GA1*S1**2)
      S2=D2-((D2-D3)*S1)/CN55
      CALL SSWTCH(1,J)
      GO TO (6222,6223),J
 6222 WRITE(61,119)S1
  119 FORMAT(40X,30H*ITERATION FOR EQUATION 1276= E17.10)
 6223 IF(ABS(S1-S2)-TOL1)120,120,121
  121 S1=S2
      GO TO 118
  120 Q=(S2**2-1.)/GA1
      FC=1.-((D2)/SQRT(1.+GA1*Q))+((D2-D3)/SQRT(1.+GA2*Q))
      WRITE(61,241)FC
      CTST=SQRT(Q)
      WRITE(61,173)Q,CTST
      CN56=1./CTST
      THETS=ATAN(CN56)
      THETM=(ACOS(V0/V3))*57.29577951
      XF=(2.*Y1/(AK1-1.))*1./CTST*(1.-D2*SQRT(1.+GA1*Q)+(D2-D3)*SQRT(1.+
     1GA2*Q))
      SCNDD=((2.*V0*(1.+Q)**2)/(SMU1*CTST))*((D2-D3)*GA2/(1.+GA2*Q)**1.5
     1-D2*GA1/(1.+GA1*Q)**1.5)
      OMEGE=4.*CTST/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      GO TO 100
   91 TST1=Y2/Y1
      WRITE(61,9111)TST1
 9111 FORMAT(/33X,6HY2/Y1=E17.10)
      IF(D2-TST1)122,123,124
  124 WRITE(61,6253)
 6253 FORMAT(57X,11HSUBDOMAIN D)
      IF(SMU2)125,126,125
  126 WRITE(61,127)
  127 FORMAT(/52X,18H**** NO FOCUS ****)
      GO TO 16
  125 S1=D2-SQRT(GA1/GA2)*(D2-D3)
  128 CN65=SQRT((1.-GA2/GA1)+GA2/GA1*S1**2)
      S2=D2-((D2-D3)*S1)/CN65
      CALL SSWTCH(1,J)
      GO TO (6224,6225),J
 6224 WRITE(61,129)S1
  129 FORMAT(40X,30H*ITERATION FOR EQUATION 1600= E17.10)
 6225 IF(ABS(S1-S2)-TOL1)130,130,131
  131 S1=S2
      GO TO 128
  130 Q=(S2**2-1.0)/GA1
      FC=1.-((D2)/SQRT(1.+GA1*Q))+((D2-D3)/SQRT(1.+GA2*Q))
      WRITE(61,241)FC
      CTST=SQRT(Q)
      WRITE(61,173)Q,CTST
      CN87=1./CTST
      THETS=ATAN(CN87)
      THETM=(ACOS(V0/V3))*57.29577951
      XF=(2.*Y1/(AK1-1.))*1./CTST*(1.-D2*SQRT(1.+GA1*Q)+(D2-D3)*SQRT(1.+
     1GA2*Q))
```

```
      SCNDD=((2.*V0*(1.+Q)**2)/(SMU1*CTST))*((D2-D3)*GA2/(1.+GA2*Q)**1.5
     1-D2*GA1/(1.+GA1*Q)**1.5)
      OMEGE=4.*CTST/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      GO TO 18
  123 CO1=SMU2/SMU3
      CO2=Y2/(Y3-Y2)
      IF(CO1-CO2)132,132,133
  133 WRITE(61,127)
      GO TO 16
  132 CO3=(1.-((Y2-Y1)/Y2)*SMU2/SMU3)**2
      Q=1./GA1*(1./CO3-1.0)
      XF=2.*(AK1+1.)*Y2*SQRT(Q/(1.+GA1*Q))
      CTST=SQRT(Q)
      FC=1.-((D2)/SQRT(1.+GA1*Q))+((D2-D3)/SQRT(1.+GA2*Q))
      WRITE(61,241)FC
      WRITE(61,173)Q,CTST
      CN95=1.0/CTST
      THETS=ATAN(CN95)
      THETM=(ACOS(V0/V3))*57.29577951
      SCNDD=-(2.*V0*D2*GA1*(1.+Q)**2)/(SMU1*CTST*(1.+GA1*Q)**1.5)
      OMEGE=4.*CTST/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      GO TO 18
  122 WRITE(61,6252)
 6252 FORMAT(57X,11HSUBDOMAIN C)
      CCO3=(D2**2/(1.0-GA1/GA2))**0.3333333333
      CCO4=((D2-D3)**2/(1.0-GA2/GA1))**0.3333333333
      CE= CCO3+CCO4
      WRITE(61,1221)CE
 1221 FORMAT(/33X,6HCE=    E17.10)
      IF(CE-1.0)136,135,134
  134 WRITE(61,127)
      GO TO 16
  135 CO4=(D2*(1.-GA1/GA2))**0.6666666667-1.0
      Q=CO4/GA1
      CTST=SQRT(Q)
      WRITE(61,173)Q,CTST
      CN75=1.0/CTST
      THETS=ATAN(CN75)
      THETM=(ACOS(V0/V3))*57.29577951
      XF=(2.*Y1/(AK1-1.))*1./CTST*(1.-D2*SQRT(1.+GA1*Q)+(D2-D3)*SQRT(1.+
     1GA2*Q))
      SCNDD=((2.*V0*(1.+Q)**2)/(SMU1*CTST))*((D2-D3)*GA2/(1.+GA2*Q)**1.5
     1-D2*GA1/(1.+GA1*Q)**1.5)
      OMEGE=4.*CTST/(XF*SCNDD)
      AIL=ABS(OMEGE)*XF*XF
      GO TO 18
  136 J1P=0
      J2P=1
      S1=(D2/SQRT(1.-GA1/GA2)-1.0)/(D2-D3)
  137 CO6=SQRT((1.-GA1/GA2)*S1**2+GA1/GA2)
      S2=(D2/(D2-D3))*S1/CO6-1./(D2-D3)
      CALL SSWTCH(1,J)
      GO TO (6226,6227),J
```

```
6226 WRITE(61,138)S1
 138 FORMAT(40X,30H*ITERATION FOR EQUATION 1475= E17.10)
6227 IF(ABS(S1-S2)-TOL1)139,139,140
 140 S1=S2
     GO TO 137
 139 Q=1./GA2*(1./S2**2-1.)
     FC=1.-((D2)/SQRT(1.+GA1*Q))+((D2-D3)/SQRT(1.+GA2*Q))
     WRITE(61,241)FC
     CTST=SQRT(Q)
     WRITE(61,173)Q,CTST
     CN77=1.0/CTST
     THETS=ATAN(CN77)
     THETM=(ACOS(V0/V3))*57.29577951
     XF=(2.*Y1/(AK1-1.))*1./CTST*(1.-D2*SQRT(1.+GA1*Q)+(D2-D3)*SQRT(1.+
    1GA2*Q))
     SCNDD=((2.*V0*(1.+Q)**2)/(SMU1*CTST))*((D2-D3)*GA2/(1.+GA2*Q)**1.5
    1-D2*GA1/(1.+GA1*Q)**1.5)
     OMEGE=4.*CTST/(XF*SCNDD)
     AIL=ABS(OMEGE)*XF*XF
     GO TO 100
 105 J1P=4
     S1=1.+(D3-1.)/(1.+(D2-D3)*(1.-GA2/GA1))
 141 CO7=SQRT((1.-GA2/GA1)+(GA2/GA1)*S1**2)
     S2=D2-((D2-D3)*S1)/CO7
     CALL SSWTCH(1,J)
     GO TO (6228,6229),J
6228 WRITE(61,142)S1
 142 FORMAT(40X,30H*ITERATION FOR EQUATION 1476= E17.10)
6229 IF(ABS(S1-S2)-TOL1)143,143,144
 144 S1=S2
     GO TO 141
 143 Q=(S2**2-1.)/GA1
     FC=1.-((D2)/SQRT(1.+GA1*Q))+((D2-D3)/SQRT(1.+GA2*Q))
     WRITE(61,241)FC
     CTST=SQRT(Q)
     WRITE(61,173)Q,CTST
     CN91=1.0/CTST
     THETS=ATAN(CN91)
     THETM=(ACOS(V0/V3))*57.29577951
     XF=(2.*Y1/(AK1-1.0))*1./CTST*(1.-D2*SQRT(1.+GA1*Q)+(D2-D3)*SQRT(1.
    1+GA2*Q))
     SCNDD=((2.*V0*(1.+Q)**2)/(SMU1*CTST))*((D2-D3)*GA2/(1.+GA2*Q)**1.5
    1-D2*GA1/(1.+GA1*Q)**1.5)
     OMEGE=4.*CTST/(XF*SCNDD)
     AIL=ABS(OMEGE)*XF*XF
     GO TO 100
  16 Y3=Y3+DY3
  37 CONTINUE
     Y2=Y2+DY2
  36 CONTINUE
     Y1=Y1+DY1
  35 CONTINUE
     Y0=Y0+DY0
  34 CONTINUE
     V3=V3+DV3
```

```
   33 CONTINUE
      V2=V2+DV2
   32 CONTINUE
      V1=V1+DV1
   31 CONTINUE
      V0=V0+DV0
   30 CONTINUE
      IF(LAST)1000,1001,1000
 1001 STOP
      END
         FINIS
```

APPENDIX II

AVERAGE INTENSITY LEVEL IN THE NEIGHBORHOOD OF A FOCUS

From Reference 1, pp. 34-36, it is seen that the mean density, ρ_m, over a small stretch of the x-axis is

$$\rho_m = \frac{\bar{\rho}\,\bar{r}^2}{\epsilon}\,|\omega|, \qquad \text{(A-1)}$$

where $\bar{\rho}\,\bar{r}^2$ is related to the energy output of the sound source and ω is a small quantity dependent on the atmospheric parameters. The quantity $\bar{\rho}\,\bar{r}^2$ (from Ref. 4) is equal to

$$\frac{P}{2\pi}\,\frac{1}{V_o},$$

where P is the total acoustic power of the source and V_o is the propagation velocity of sound at the sound source. Upon choosing an initial reference intensity,

$$I_o = 10^{-12}\,\frac{\text{watts}}{m^2},$$

it follows that the average intensity level in the ϵ neighborhood of a focus may be given by

$$\text{I.L.} = D - 18 + 10\,\log_{10}|\omega| - 10\,\log_{10}\epsilon \text{ db re } 10^{-12}\,\text{watts/m}^2, \qquad \text{(A-2)}$$

where D is the total power level of the sound source in db re 10^{-13} watts.

Equation (A-2), then, enables one to compute the average intensity level quite readily. Also, it leaves one free to choose not only the power output of the source but also the neighborhood about the focus over which he wishes to know the average intensity level. It must be emphasized here that ϵ must not be chosen too large, since the expression for ω is derived for an infinitesimal neighborhood.

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FOCAL POINT COMPUTATION IN A THREE-LAYERED ATMOSPHERE

By

James E. Mabry

TECHNICAL AND SCIENTIFIC STAFF
AERO-ASTRODYNAMICS LABORATORY

APPROVAL TM X-53479

FOCAL POINT COMPUTATION IN A THREE-LAYERED ATMOSPHERE

By James E. Mabry

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This document has also been reviewed and approved for technical accuracy.

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Scientific Assistant to Director
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for E. D. Geissler

E. D. Geissler
Director, Aero-Astrodynamics Laboratory

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